Feshbach Resonances and Medium Effects in ultracold atomic Gases *

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Abstract

We develop an effective low energy theory for multi-channel scattering of cold atomic alkali atoms with particular focus on Feshbach resonances. The scattering matrix is expressed in terms of observables only and the theory allows for the inclusion of many-body effects both in the open and in the closed channels. We then consider the frequency and damping of collective modes for Fermi gases and demonstrate how medium effects significantly increase the scattering rate determining the nature of the modes. Our results obtained with no fitting parameters are shown to compare well with experimental data.

1 Introduction

The study of cold atomic gases has now been at the forefront of low temperature physics for more than a decade. One can manipulate these gases with impressive experimental flexibility using the powerful tools of quantum optics. This has produced a string of ground breaking results relevant across many fields of physics including quantum optics, AMO and condensed matter physics [1, 2]. A particularly attractive feature of cold atomic gases is the ability to manipulate the atom-atom interaction with the use of Feshbach resonances. The interaction can be made strong/weak and attractive/repulsive simply by tuning an external magnetic field. This has resulted in many important discoveries concerning strongly interacting many-body systems and the pace at which new results are being reported shows no sign of slowing down.

Sophisticated and very precise coupled channels calculations have been developed to describe atomic Feshbach resonances at the two-body level [3]. Such coupled channels approaches are in general not easily generalized to study the

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intriguing many-body effects observed in the atomic gases. Several effective theories have therefore been developed which include a simplified version of the two-body Feshbach physics such that many-body calculations are tractable [2]. Most of these theories either neglect the Feshbach molecule entirely using so-called single channel models [4] or put it in by hand as a point boson [5]. Such approaches have been very successful in calculating various many-body properties of the atomic gases for wide resonances where the multi-channel nature of the scattering is less important. For narrow resonances however, single channel approximations cannot be expected to be accurate, and even for wide resonances there are observables which depend specifically on the multi-channel nature of the scattering.

To address this, we describe in this paper an effective theory for the Feschbach scattering which in the spirit of Landau expresses the multi-channel scattering matrix in terms of observables only. The Feshbach molecule emerges dynamically as a proper two-body state, yet the theory is still simple enough to be easily generalized to treat many-body effects. As an application of this theory, we consider the collective modes of trapped atomic Fermi gases. The study of collective modes is a powerful probe into the properties of interacting quantum liquids. In cold atomic Fermi gases, collective mode spectroscopy has revealed a wealth of information about zero temperature $T = 0$ [2] as well as $T > 0$ properties [6, 7, 8]. We outline how one can calculate the frequency and damping of the collective modes in the normal phase above the critical temperature T_c for superfluidity. Focus is on how the modes reveal information about the collisional properties and many-body effects.

2 Landau Theory for in-medium Scattering

First we develop an effective low energy theory for fermionic alkali atom-atom scattering in a medium. Consider alkali atoms in a magnetic field B oriented along the z-direction. The strongest part of the atom-atom interaction is the electrostatic central potential given by

$$V(r) = \frac{V_s(r) + 3\,V_t(r)}{4} + [V_t(r) - V_s(r)]\,\vec{S}_1 \cdot \vec{S}_2 \tag{1}$$

where $V_s(r)$ and $V_t(r)$ are the singlet and triplet potentials and $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spins of the valence electrons of the two alkali atoms [1]. Scattering via the potential (1) is characterized by channels of anti-symmetrized two-particle states with the same z-projection M_z of the total spin $\vec{F}$. For a given M_z, the two-particle state with the lowest energy $\epsilon_{\alpha_2} + \epsilon_{\alpha_1}$ constitutes the open channel $|o\rangle = |\alpha_1, \alpha_2\rangle$. Here $\hat{H}_{\rm spin}|\alpha\rangle = \epsilon_\alpha|\alpha\rangle$ are the eigenstates of the single particle hyperfine Hamiltonian [1]. The interaction (1) couples this channel to a number of higher energy states which form a set of closed channels $|c^{(n)}\rangle = |\alpha_3^{(n)}, \alpha_4^{(n)}\rangle$. The threshold energies for the closed channels are then $E_{\rm th}^{(n)}(B) = \epsilon_{\alpha_4^{(n)}} + \epsilon_{\alpha_3^{(n)}} - \epsilon_{\alpha_2} - \epsilon_{\alpha_1}$ and they depend on the magnetic field.

Focus now on the case where there is one open $|o\rangle$ and one closed scattering channel $|c\rangle$. It should be emphasized that our effective theory is readily generalized to more than two channels if appropriate. To arrive at an effective theory for the scattering, we want to eliminate the bare microscopic interaction (1) which has a complicated momentum dependence. The high energy physics is eliminated by introducing an effective interaction U_{ij} which is a solution to the zero energy Lippmann-Schwinger equation when the hyperfine splitting of the channels is ignored. This results in a momentum independent low energy interaction given by [9]

$$\hat{U}(\mathbf{q}', \mathbf{q}) = \frac{4\pi}{m}\left[\frac{a_s + 3a_t}{4} + (a_t - a_s)\mathbf{S}_1 \cdot \mathbf{S}_2\right] \tag{2}$$

where a_s and a_t are the scattering lengths for the singlet $V_s(r)$ and triplet $V_t(r)$ potentials, respectively. Any finite range effects can be introduced through form factors which we have suppressed here for clarity. Using this low energy interaction, the Lippmann-Schwinger equation reduces to a simple 2×2 matrix equation

$$\begin{bmatrix} T_{cc} & T_{co} \\ T_{oc} & T_{oo} \end{bmatrix}^{-1} = \begin{bmatrix} U_{cc} & U_{co} \\ U_{oc} & U_{oo} \end{bmatrix}^{-1} - \begin{bmatrix} \Pi_c & 0 \\ 0 & \Pi_o \end{bmatrix} \tag{3}$$

where $T_{ij}(\omega, \vec{K})$ is the scattering matrix between the channels i and j. In addition to the usual dependence on the energy ω, it also depends on the center-of-mass momentum $\vec{K}$ since Galilean invariance is broken by the presence of the medium. The expressions for the pair propagators in the open and closed channels $\Pi_0(\omega, \vec{K})$ and $\Pi_c(\omega, \vec{K}, B)$ with medium effects included through the ladder approximation are given in Ref. [9]. Equation (3) is easily solved and the open channel scattering matrix can be written as

$$T_{oo} = \frac{U_{oo}}{1 - U_{oo}\Pi_o} + \frac{U_{oc}}{1 - U_{oo}\Pi_o} D \frac{U_{co}}{1 - U_{oo}\Pi_o} \tag{4}$$

where

$$D^{-1}(\mathbf{K}, \omega) = {\Pi_c}^{-1} - U_{cc} - {U_{oc}}^2 \frac{\Pi_o}{1 - U_{oo}\Pi_o} \tag{5}$$

is the in-medium pair propagator in the closed channel. Equation (4) provides a transparent physical interpretation of the multi channel scattering: The first term in (4) describes scattering induced by the open channel interaction only and the second term describes the scattering via the closed channel. The diagrammatic structure of (4)-(5) is shown in Fig. 1.

The scattering of alkali atoms depends on the magnetic field B both through the hyperfine states and the matrix elements U_{ij}. Close to a Feshbach resonance located at a given field B_0, the zero energy two-body scattering matrix in the open channel can be parametrized as

$$T_{oo}^{vac} \equiv \frac{4\pi a}{m} = \frac{4\pi a_{\rm bg}}{m}\left(1 - \frac{\Delta B}{B - B_0}\right). \tag{6}$$

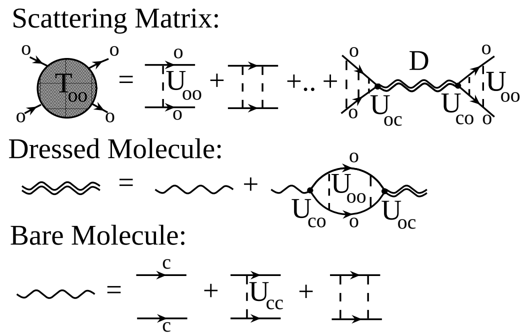

Figure 1: The scattering matrix (4) decomposed into scattering in the open and closed channels. The closed channel molecule is dressed via its coupling to the open channel. Fermions are indicated by straight lines, interaction within a channel is indicated by dashed lines, and coupling between the open and closed channels is indicated by •.

Here, $a_{\rm bg}$ is the (non-resonant) background scattering length and ΔB the width of the resonance. The Feshbach resonance comes from the presence of a molecular state in the closed channel. It is thus contained in the second term in (4). The energy $\omega_{\mathbf{K}}$ of a Feshbach molecule (including medium effects) with momentum $\mathbf{K}$ is determined by $D^{-1}(\mathbf{K},\omega_{\mathbf{K}})=0$. By making a pole expansion of (4) around $B=B_0$ and comparing with (6), one can write the scattering matrix in the very useful form [9]

$$T_{oo}=\frac{T_{\rm bg}}{\left(1+\frac{\Delta\mu\Delta B}{\tilde{\omega}+h(\omega)-\Delta\mu(B-B_0)}\right)^{-1}-T_{\rm bg}\Pi_o(\omega)}. \tag{7}$$

Here $T_{\rm bg}=4\pi a_{bg}/m$, $\tilde{\omega}=\omega-K^2/4m$, and $\Delta\mu$ is the magnetic moment of the Feshbach molecule with respect to the open channel. A detailed analysis of the molecular propagator (5) shows that $\Delta\mu$ can be split into a contribution from the magnetic dependence of the bare closed channel state and a contribution from screening due to coupling to high energy states in the open channel. This screening which reduces the magnetic moment from its bare value is often ignored in the literature. It comes from a *linear* frequency dependence of the molecule self energy in addition to the well known $\sqrt{\omega}$ threshold dependence, and it can lead to a significant reduction of the magnetic moment of the molecule [9, 10]. The function $h(\omega)$ is given in Ref. [9]. It describes effects coming from the composite two-fermion nature of the Feshbach molecule, and it is here that many-body effects in the closed channel enter.

With (7), we have arrived at an effective low energy theory for scattering in a medium. The complicated energy and momentum dependent multichannel scattering matrix is expressed in a simple way through the physical observ-

ables $a_{\rm bg}$, B_0, ΔB, and $\Delta\mu$. The parameters $a_{\rm bg}$, B_0, ΔB can be measured in scattering experiments whereas the magnetic moment of the Feshbach molecule can be measured in rethermalization experiments [9]. Contrary to many other approaches in the literature, the theory allows one to include non-trivial many-body effects in the closed channel as well as in the open channel. Examples of such closed channel medium effects were considered in Ref. [9].

3 Collective Modes and Viscous Damping

We now consider the collective modes of trapped Fermi gases and examine how they can reveal information about the scattering properties discussed in the previous section. We focus on the normal state for temperatures $T \geq T_c$. The dynamics of the gas is assumed to be described by a semiclassical distribution function $f(\mathbf{r},\mathbf{p},t)$ which satisfies the Boltzmann equation. A collective mode corresponds to a deviation $\delta f = f - f^0$ away from the equilibrium distribution $f^0(\mathbf{r},\mathbf{p})$. By expanding δf in a set of basis states with the symmetry appropriate for the particular mode considered, one can express the Boltzmann equation in matrix form [8, 11]. The corresponding determinants determine the mode frequency ω.

To be specific, we model the collective modes studied experimentally in Ref. [8], where the atoms are trapped in a very elongated harmonic potential of the form $V(\mathbf{r}) = m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)/2$ with $\omega_z \ll \omega_y \leq \omega_x$. The motion of the collective modes is then mainly in the xy-plane. For the scissors mode, the determinant equation determining the mode frequency becomes [12]

$$\frac{i\omega}{\tau}(\omega^2 - \omega_h^2) + (\omega^2 - \omega_{c1}^2)(\omega^2 - \omega_{c2}^2) = 0. \tag{8}$$

Here $\omega_h = \sqrt{\omega_x^2 + \omega_y^2}$ is the mode frequency in the hydrodynamic limit when $\omega\tau \ll 1$ characteristic of many collisions, and $\omega_{c1} = \omega_x + \omega_y$ and $\omega_{c2} = |\omega_x - \omega_y|$ are the mode frequencies in the collisionless limit $\omega\tau \gg 1$ [13]. The collision rate $1/\tau$ is

$$\frac{1}{\tau} = \frac{\int d^3rd^3pp_xp_yI[p_xp_y]}{\int d^3rd^3pp_x^2p_y^2f^0(1-f^0)} \tag{9}$$

where $I[p_xp_y]$ is the collision integral in the Boltzmann equation weighted by the momentum function p_xp_y [8, 11]. It is in the collision integral, that the scattering matrix enters. The collision rate (9) is closely related to the viscosity of the gas and it is therefore sometimes called the viscous relaxation rate [14].

When the atoms are strongly interacting, there are significant pair correlations even in the normal phase. The correlations depend strongly on temperature and interaction strength which is parametrized by the scattering length a in (6). Correlations and their dependence on a and T enter the theory for collective modes through (9). In Fig. 2, we plot the scattering rate $1/\tau$ as a function of T for: (a) strong coupling right at a Feshbach resonance $|a| \to \infty$, and (b) in the weak coupling regime $k_Fa = -0.06$. To clearly identify the importance of

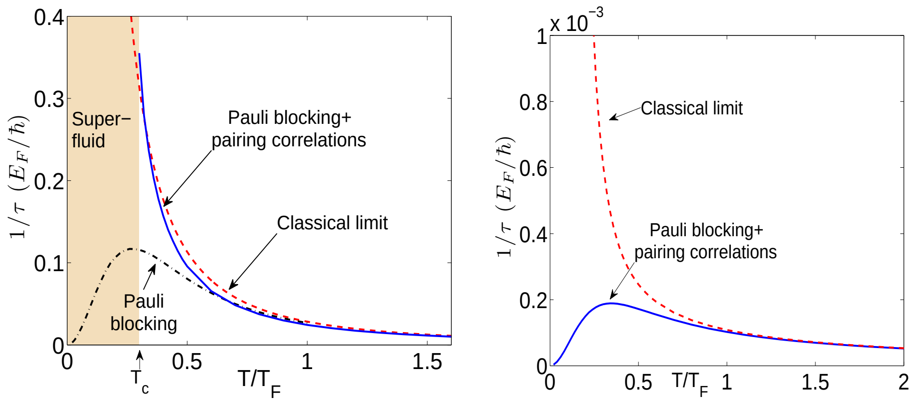

Figure 2: The viscous relaxation rate rate $1/\tau$ for a gas in the unitarity limit (a) (From [8]) and in the weak coupling limit (b). The superfluid region is indicated in (a) whereas it is not visible in (b) due to the smallness of T_c. The dashed lines show the classical limit, the dash-dotted lines include Fermi blocking, and the solid lines include medium effects in the cross section.

medium effects on the scattering matrix, the rate is calculated using three different approximations. The dashed curves are a classical approximation where Pauli blocking effects are neglected and the two-body scattering matrix is used. The dash-dotted lines include Pauli blocking in the collision integral $I[p_x p_y]$ while the two-body scattering matrix is still used. We see that Pauli blocking reduces the scattering rate as compared to the classical result as expected; the classical rate scales as T^{-2} whereas $\tau^{-1} \propto T^2$ for $T \to 0$ due to Pauli blocking. Finally, the solid lines use the many-body scattering matrix (7) in addition to including Pauli blocking effects in the collision integral. Medium effects in the scattering matrix are included through the pair propagator $\Pi_0(\omega, \vec{K})$ in (7). As seen by comparing the solid and the dash-dotted lines in Fig. 2 (a), medium effects significantly increase the scattering rate over a wide range of temperatures above T_c for the strong coupling case. This is due to pair correlations. It is the same physics which gives rise to a divergence in the $\vec{K} = 0$ scattering matrix at T_c signaling the onset of Cooper pairing. From Fig. 2 (a), we see that the scattering rate calculated including both pair correlations and Pauli blocking effects is almost the same as the classical rate which neglects both effects. Thus, pair correlations nearly cancel the reduction of the scattering rate due to Pauli blocking in the normal phase. These strong pair correlations are often referred to as the pseudogap effect. So we have demonstrated that it is essential to include medium effects in the scattering matrix (7) when one considers strong coupling Fermi gases; a simple two-body scattering matrix strongly underestimates the correlations. In contrast, there are no observable medium effects on the scattering matrix in the weak coupling regime depicted in Fig. 2 (b). Here the curves using a two-body and a many-body T_{oo} are essentially

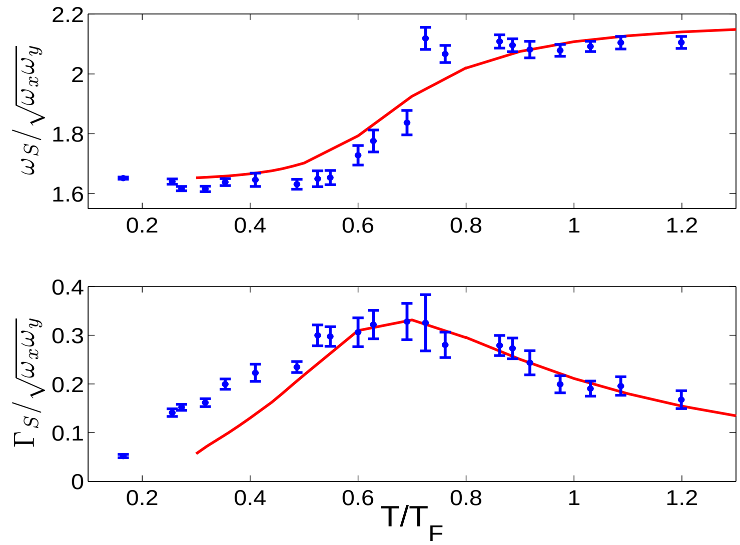

Figure 3: The frequency ω_S and damping Γ_S of the scissors mode. The solid line is theory and the experimental points are from Ref. [8].

indistinguishable.

Once we know the scattering rate $1/\tau$, we can calculate the collective mode frequencies as discussed above. In Fig. 3, we plot the scissors mode frequency ω_S and damping Γ_S obtained from the real and imaginary parts of the solution of (8), i.e. $\omega = \omega_S - i\Gamma_S$. The scattering rate is obtained from (9) using the many-body scattering matrix (7). The gas is strongly interacting with $|a| \to \infty$ and we compare with the experimental data in Ref. [8]. Taking into account the experimental uncertainties and the fact that there are *no fitting parameters* in the theory, the agreement between theory and experiment is good. This indicates that the expressions (7) and (9) account for most of the correlation effects even for strongly correlated Fermi gases. Since the medium effects increase the scattering rate significantly, they make the modes more hydrodynamic. We conclude that the observation of well defined hydrodynamic modes above T_c (See Fig. 3 and Ref. [8]) is a signature of many-body effects on the scattering.

4 Conclusion

We developed an effective theory for multi-channel Feshbach scattering in cold alkali atom gases. The theory expresses the scattering in terms of physical observables only. It allows for the inclusion of many-body effects in all channels and provides a precise link between microscopic two-body multi-channel calculations and effective many-body theories. Many-body effects significantly increase the scattering rate over wide range of temperatures. We showed how

this can be detected on the frequency and damping of collective modes. Our results were finally compared to experimental data obtaining good agreement.

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